From the 2017 Administration

AP® CollegeBoard

Calculus AB

Practice Exam

This exam may not be posted on school or personal websites, nor electronically redistributed for any reason. This Released Exam is provided by the College Board for AP Exam preparation. Teachers are permitted to download the materials and make copies to use with their students in a classroom setting only. To maintain the security of this exam, teachers should collect all materials after their administration and keep them in a secure location.

Further distribution of these materials outside of the secure College Board site disadvantages teachers who rely on uncirculated questions for classroom testing. Any additional distribution is in violation of the College Board's copyright policies and may result in the termination of Practice Exam access for your school as well as the removal of access to other online services such as the AP Teacher Community and Online Score Reports.

AP Central is the official online home for the AP Program: apcentral.collegeboard.org

AP® Calculus AB Exam

SECTION I: Multiple Choice

2017

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
30
Time
1 hour
Electronic Device
None allowed

Part B

Number of Questions
15
Time
45 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 30 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 90 on page 3 of the answer sheet. Because Part A and Part B offer only four answer options for each question, do not mark the (E) answer circle for any question. The survey questions are numbers 91 through 94.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Form I
Form Code 4NBP4-S

66

CALCULUS AB

SECTION I, Part A

Time—1 hour

Number of questions—30

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. If $f(x) = \left(2x^2 + 5\right)^7$, then $f'(x) =$

   (A) $7(4x)^6$

   (B) $7\left(2x^2 + 5\right)^6$

   (C) $14x^2\left(2x^2 + 5\right)^6$

   (D) $28x\left(2x^2 + 5\right)^6$

---

2. $\displaystyle\int \frac{1}{3x + 12}\, dx =$

   (A) $-3 \ln |x + 4| + C$

   (B) $\dfrac{1}{3} \ln |x + 4| + C$

   (C) $\ln |x + 4| + C$

   (D) $3 \ln |x + 4| + C$

GO ON TO THE NEXT PAGE.

3. If $f(x) = \dfrac{5 - x}{x^3 + 2}$, then $f'(x) =$

(A) $\dfrac{-4x^3 + 15x^2 - 2}{\left(x^3 + 2\right)^2}$

(B) $\dfrac{-2x^3 + 15x^2 + 2}{\left(x^3 + 2\right)^2}$

(C) $\dfrac{2x^3 - 15x^2 - 2}{\left(x^3 + 2\right)^2}$

(D) $\dfrac{4x^3 - 15x^2 + 2}{\left(x^3 + 2\right)^2}$

GO ON TO THE NEXT PAGE.

t	0	0.5	2	3
$v(t)$	20	60	40	30

4. The table above gives the velocity $v(t)$, in miles per hour, of a truck at selected times t, in hours. Using a trapezoidal sum with the three subintervals indicated by the table, what is the approximate distance, in miles, the truck traveled from time $t = 0$ to $t = 3$?

   (A) 140 (B) 130 (C) 125 (D) 120

---

5. If $f(x) = \sin(x^2 + \pi)$, then $f'(\sqrt{2\pi}) =$

   (A) $-2\sqrt{2\pi}$ (B) -2 (C) -1 (D) $\cos(2\sqrt{2\pi})$

GO ON TO THE NEXT PAGE.

6. If f is the function given by $f(x) = 3x^2 - x^3$, then the average rate of change of f on the closed interval $[1, 5]$ is

(A) -21 (B) -13 (C) -12 (D) -9

---

7. If $\int_4^{-10} g(x)\,dx = -3$ and $\int_4^6 g(x)\,dx = 5$, then $\int_{-10}^6 g(x)\,dx =$

(A) -8 (B) -2 (C) 2 (D) 8

GO ON TO THE NEXT PAGE.

8. If f is the function given by $f(x) = e^{x/3}$, which of the following is an equation of the line tangent to the graph of f at the point $(3 \ln 4, 4)$?

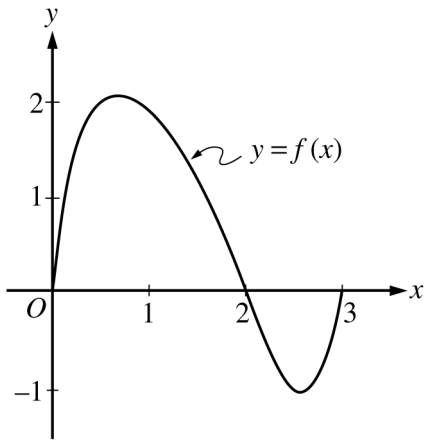
(A) $y - 4 = \frac{4}{3}(x - 3 \ln 4)$

(B) $y - 4 = 4(x - 3 \ln 4)$

(C) $y - 4 = 12(x - 3 \ln 4)$

(D) $y - 3 \ln 4 = 4(x - 4)$

GO ON TO THE NEXT PAGE.



9. The graph of a function f is shown above. Which of the following expresses the relationship between $\int_0^2 f(x)\,dx$, $\int_0^3 f(x)\,dx$, and $\int_2^3 f(x)\,dx$?

(A) $\int_0^2 f(x)\,dx < \int_0^3 f(x)\,dx < \int_2^3 f(x)\,dx$

(B) $\int_0^3 f(x)\,dx < \int_0^2 f(x)\,dx < \int_2^3 f(x)\,dx$

(C) $\int_2^3 f(x)\,dx < \int_0^2 f(x)\,dx < \int_0^3 f(x)\,dx$

(D) $\int_2^3 f(x)\,dx < \int_0^3 f(x)\,dx < \int_0^2 f(x)\,dx$

GO ON TO THE NEXT PAGE.

10. $\int_0^2 \left(x^3 + 1\right)^{1/2} x^2 \, dx =$

(A) $\frac{52}{9}$ (B) 6 (C) $\frac{26}{3}$ (D) $\frac{52}{3}$

---

11. If $x^2 + xy - 3y = 3$, then at the point $(2, 1)$, $\frac{dy}{dx} =$

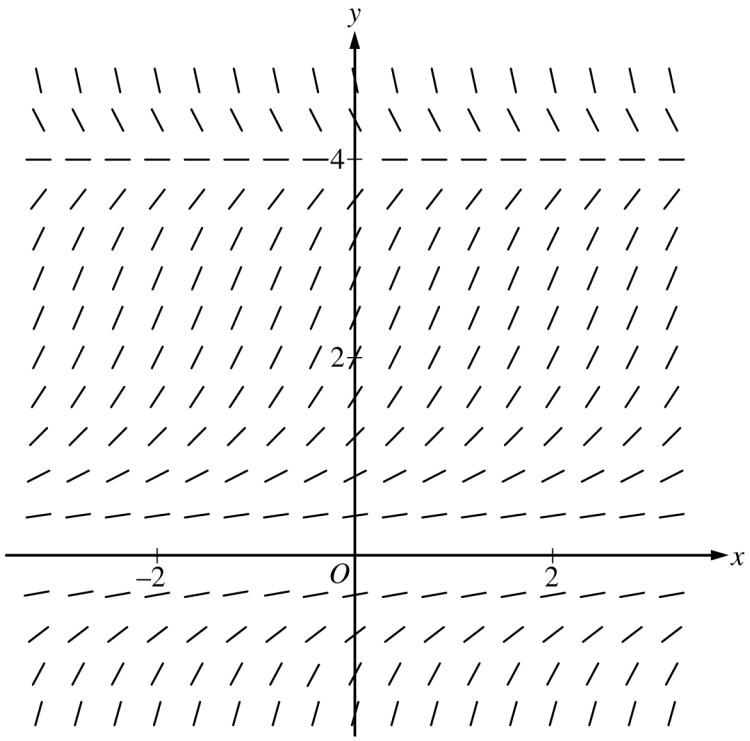
(A) 5 (B) 4 (C) $\frac{7}{3}$ (D) 2

GO ON TO THE NEXT PAGE.

12. The number of gallons of water in a storage tank at time t, in minutes, is modeled by $w(t) = 25 - t^2$ for $0 \le t \le 5$. At what rate, in gallons per minute, is the amount of water in the tank changing at time $t = 3$ minutes?

(A) 66 (B) 16 (C) -3 (D) -6

GO ON TO THE NEXT PAGE.



13. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x(4-y)}{4}$

(B) $\frac{dy}{dx} = \frac{y(4-y)}{4}$

(C) $\frac{dy}{dx} = \frac{xy(4-y)}{4}$

(D) $\frac{dy}{dx} = \frac{y^2(4-y)}{4}$

GO ON TO THE NEXT PAGE.

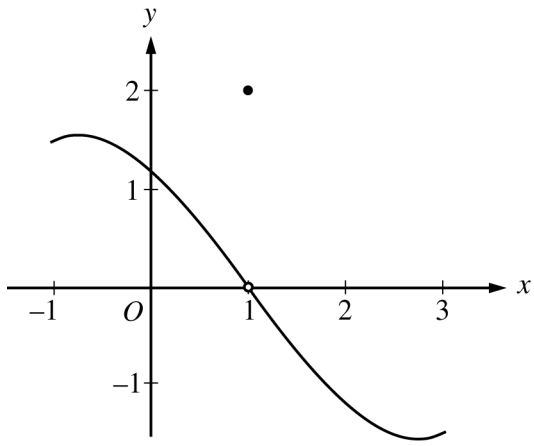
14. The weight of a population of yeast is given by a differentiable function y, where $y(t)$ is measured in grams and t is measured in days. The weight of the yeast population increases according to the equation $\frac{dy}{dt} = ky$, where k is a constant. At time $t = 0$, the weight of the yeast population is 120 grams and is increasing at the rate of 24 grams per day. Which of the following is an expression for $y(t)$?

(A) $120e^{24t}$

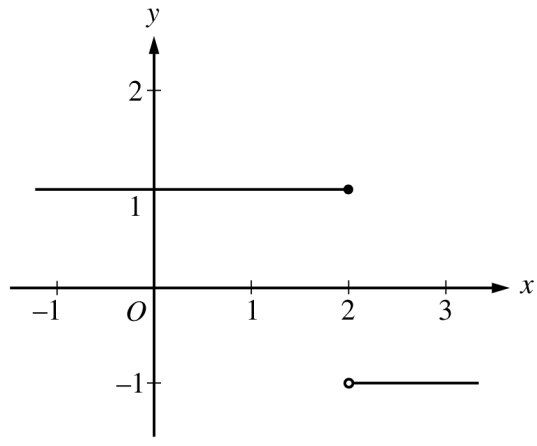
(B) $120e^{t/5}$

(C) $e^{t/5} + 119$

(D) $24t + 120$



Graph of f



Graph of g

15. The graphs of the functions f and g are shown in the figures above. Which of the following statements is false?

(A) $\lim_{x \to 1} f(x) = 0$

(B) $\lim_{x \to 2} g(x)$ does not exist.

(C) $\lim_{x \to 1} (f(x)\, g(x+1))$ does not exist.

(D) $\lim_{x \to 1} (f(x+1)\, g(x))$ exists.

16. Let f be the function defined by $f(x) = -3 + 6x^2 - 2x^3$. What is the largest open interval on which the graph of f is both concave up and increasing?

(A) $(0, 1)$ (B) $(1, 2)$ (C) $(0, 2)$ (D) $(2, \infty)$

---

17. A particle moves along the x-axis so that at time $t > 0$ its position is given by $x(t) = 12e^{-t}\sin t$. What is the first time t at which the velocity of the particle is zero?

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) π

GO ON TO THE NEXT PAGE.

18. Let F be the function given by $F(x) = \int_3^x (\tan(5t)\sec(5t) - 1)\, dt$. Which of the following is an expression for $F'(x)$?

(A) $\frac{1}{5}\sec(5x) - 1$

(B) $\frac{1}{5}\sec(5x) - x$

(C) $\tan(5x)\sec(5x)$

(D) $\tan(5x)\sec(5x) - 1$

---

19. Let f be the function given by $f(x) = 2\cos x + 1$. What is the approximation for $f(1.5)$ found by using the line tangent to the graph of f at $x = \frac{\pi}{2}$?

(A) -2 (B) 1 (C) $\pi - 2$ (D) $4 - \pi$

GO ON TO THE NEXT PAGE.

20. Let f be the function given by $f(x) = \dfrac{x-2}{2|x-2|}$. Which of the following is true?

(A) $\lim\limits_{x \to 2} f(x) = \dfrac{1}{2}$

(B) f has a removable discontinuity at $x = 2$.

(C) f has a jump discontinuity at $x = 2$.

(D) f has a discontinuity due to a vertical asymptote at $x = 2$.

---

21. If $f(x) = \ln x$, then $\lim\limits_{x \to 3} \dfrac{f(x) - f(3)}{x - 3}$ is

(A) $\dfrac{1}{3}$ (B) e^3 (C) $\ln 3$ (D) nonexistent

GO ON TO THE NEXT PAGE.

22. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{2y}{2x+1}$ with the initial condition $y(0) = e$ for $x > -\frac{1}{2}$?

(A) $y = e^{2x^2+2x+1}$

(B) $y = 2ex + e$

(C) $y = \sqrt{x^2 + x + e^2}$

(D) $y = \sqrt{\frac{1}{2}\ln(2x+1) + e^2}$

GO ON TO THE NEXT PAGE.

23. The region enclosed by the graphs of $y = x^2$ and $y = 2x$ is the base of a solid. For the solid, each cross section perpendicular to the y-axis is a rectangle whose height is 3 times its base in the xy-plane. Which of the following expressions gives the volume of the solid?

(A) $3\displaystyle\int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)^2 dy$

(B) $3\displaystyle\int_0^4 \left(\sqrt{y} + \frac{y}{2}\right)^2 dy$

(C) $3\displaystyle\int_0^2 (2x - x^2)^2 dx$

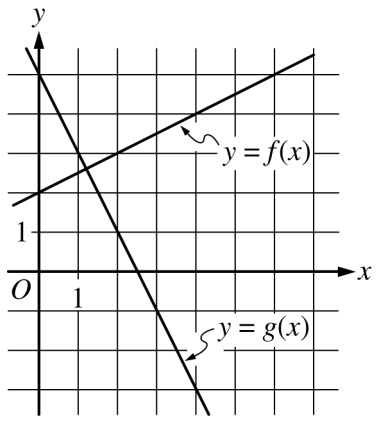
(D) $3\displaystyle\int_0^2 (2x + x^2)^2 dx$

---

24. If the average value of a continuous function f on the interval $[-2, 4]$ is 12, what is $\displaystyle\int_{-2}^{4} \frac{f(x)}{8} dx$?

(A) $\frac{3}{2}$ (B) 3 (C) 9 (D) 72

GO ON TO THE NEXT PAGE.



25. The figure above shows the graphs of the functions f and g. If $h(x) = f(x)g(x)$, then $h'(2) =$

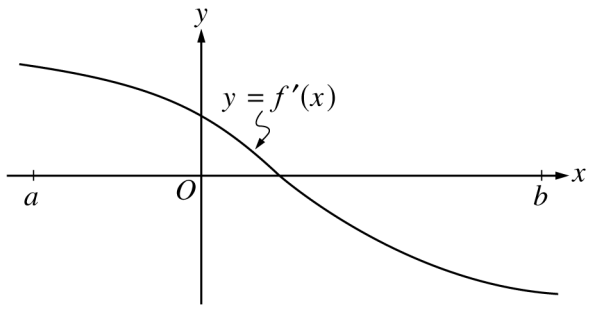
(A) $\frac{13}{2}$ (B) $\frac{1}{2}$ (C) -1 (D) $-\frac{11}{2}$

---

26. $\lim_{x \to \infty} \frac{\ln\left(e^{3x} + x\right)}{x} =$

(A) 0 (B) 1 (C) 3 (D) ∞

GO ON TO THE NEXT PAGE.



27. The graph of f', the derivative of the function f, is shown in the figure above. Which of the following statements must be true?

I. f is continuous on the open interval (a, b).

II. f is decreasing on the open interval (a, b).

III. The graph of f is concave down on the open interval (a, b).

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

GO ON TO THE NEXT PAGE.

28. An isosceles right triangle with legs of length s has area $A = \frac{1}{2}s^2$. At the instant when $s = \sqrt{32}$ centimeters, the area of the triangle is increasing at a rate of 12 square centimeters per second. At what rate is the length of the hypotenuse of the triangle increasing, in centimeters per second, at that instant?

(A) $\frac{3}{4}$ (B) 3 (C) $\sqrt{32}$ (D) 48

---

29. The graph of which of the following functions has exactly one horizontal asymptote and no vertical asymptotes?

(A) $y = \frac{1}{x^2 + 1}$

(B) $y = \frac{1}{x^3 + 1}$

(C) $y = \frac{1}{e^x - 1}$

(D) $y = \frac{1}{e^x + 1}$

GO ON TO THE NEXT PAGE.

30. For a certain continuous function f, the right Riemann sum approximation of $\int_0^2 f(x)\,dx$ with n subintervals of equal length is $\dfrac{2(n+1)(3n+2)}{n^2}$ for all n. What is the value of $\int_0^2 f(x)\,dx$?

(A) 2 (B) 6 (C) 12 (D) 20

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

PART B STARTS ON PAGE 26.

CALCULUS AB

SECTION I, Part B

Time—45 minutes

Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–90.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

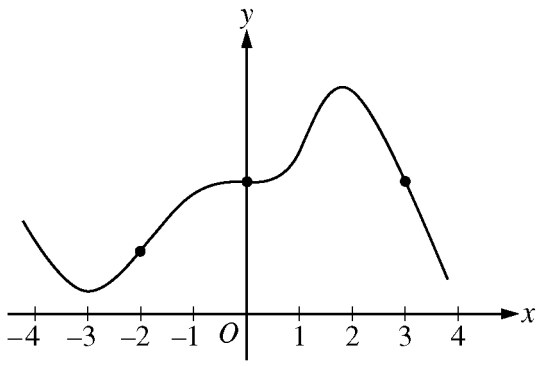
In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.



Graph of f

76. The graph of a differentiable function f is shown in the figure above. Which of the following is true?

(A) $f'(-2) < f'(0) < f'(3)$

(B) $f'(-2) < f'(3) < f'(0)$

(C) $f'(3) < f'(-2) < f'(0)$

(D) $f'(3) < f'(0) < f'(-2)$

GO ON TO THE NEXT PAGE.

77. Let $H(x)$ be an antiderivative of $\dfrac{x^3 + \sin x}{x^2 + 2}$. If $H(5) = \pi$, then $H(2) =$

(A) -9.008 (B) -5.867 (C) 4.626 (D) 12.150

---

78. The continuous function f is positive and has domain $x > 0$. If the asymptotes of the graph of f are $x = 0$ and $y = 2$, which of the following statements must be true?

(A) $\lim\limits_{x \to 0^+} f(x) = \infty$ and $\lim\limits_{x \to 2} f(x) = \infty$

(B) $\lim\limits_{x \to 0^+} f(x) = 2$ and $\lim\limits_{x \to \infty} f(x) = 0$

(C) $\lim\limits_{x \to 0^+} f(x) = \infty$ and $\lim\limits_{x \to \infty} f(x) = 2$

(D) $\lim\limits_{x \to 2} f(x) = \infty$ and $\lim\limits_{x \to \infty} f(x) = 2$

GO ON TO THE NEXT PAGE.

79. A file is downloaded to a computer at a rate modeled by the differentiable function $f(t)$, where t is the time in seconds since the start of the download and $f(t)$ is measured in megabits per second. Which of the following is the best interpretation of $f'(5) = 2.8$?

(A) At time $t = 5$ seconds, the rate at which the file is downloaded to the computer is 2.8 megabits per second.

(B) At time $t = 5$ seconds, the rate at which the file is downloaded to the computer is increasing at a rate of 2.8 megabits per second per second.

(C) Over the time interval $0 \le t \le 5$ seconds, 2.8 megabits of the file are downloaded to the computer.

(D) Over the time interval $0 \le t \le 5$ seconds, the average rate at which the file is downloaded to the computer is 2.8 megabits per second.

---

80. The function f has first derivative given by $f'(x) = x^4 - 6x^2 - 8x - 3$. On what intervals is the graph of f concave up?

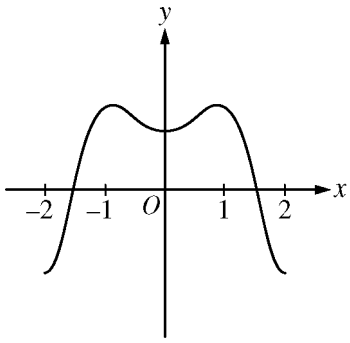
(A) $(2, \infty)$ only

(B) $(0, \infty)$

(C) $(-1, 2)$

(D) $(-\infty, -1)$ and $(3, \infty)$

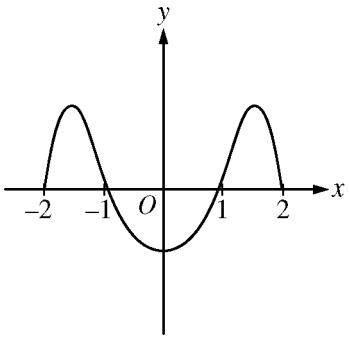
GO ON TO THE NEXT PAGE.



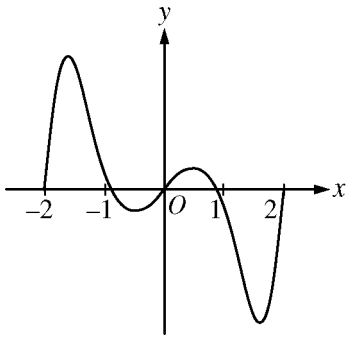
Graph of f

81. The graph of the function f is shown above for $-2 \le x \le 2$. Which of the following could be the graph of an antiderivative of f ?

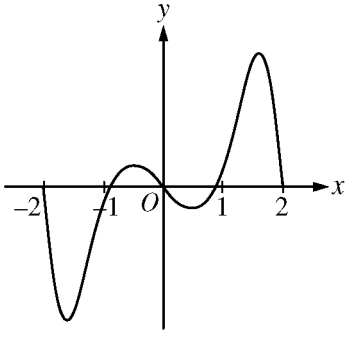
(A)



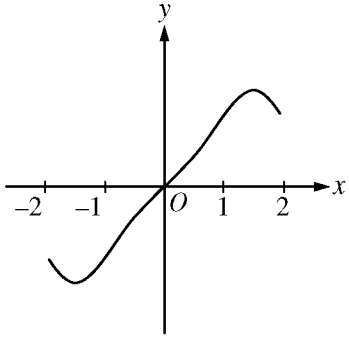
(B)



(C)



(D)



GO ON TO THE NEXT PAGE.

82. A particle travels along a straight line with velocity $v(t) = 3e^{-t/2}\sin(2t)$ meters per second. What is the total distance, in meters, traveled by the particle during the time interval $0 \le t \le 2$ seconds?

(A) 0.835 (B) 1.850 (C) 2.055 (D) 2.261

---

83. Let f be a function with derivative given by $f'(x) = \dfrac{x^3 - 8x^2 + 3}{\sqrt{x^3 + 1}}$ for $-1 < x < 9$. At what value of x does f attain a relative maximum?

(A) -0.591 (B) 0 (C) 0.638 (D) 7.953

GO ON TO THE NEXT PAGE.

84. The number of bacteria in a container increases at the rate of $R(t)$ bacteria per hour. If there are 1000 bacteria at time $t = 0$, which of the following expressions gives the number of bacteria in the container at time $t = 3$ hours?

(A) $R(3)$ (B) $1000 + R(3)$ (C) $\int_0^3 R(t)\,dt$ (D) $1000 + \int_0^3 R(t)\,dt$

---

85. The function g is continuous on the closed interval $[1, 4]$ with $g(1) = 5$ and $g(4) = 8$. Of the following conditions, which would guarantee that there is a number c in the open interval $(1, 4)$ where $g'(c) = 1$?

(A) g is increasing on the closed interval $[1, 4]$.

(B) g is differentiable on the open interval $(1, 4)$.

(C) g has a maximum value on the closed interval $[1, 4]$.

(D) The graph of g has at least one horizontal tangent in the open interval $(1, 4)$.

GO ON TO THE NEXT PAGE.

$$f''(x) = x(x-1)^2(x+2)^3$$
$$g''(x) = x(x-1)^2(x+2)^3 + 1$$
$$h''(x) = x(x-1)^2(x+2)^3 - 1$$

86. The twice-differentiable functions f, g, and h have second derivatives given above. Which of the functions f, g, and h have a graph with exactly two points of inflection?

(A) g only

(B) h only

(C) f and g only

(D) f, g, and h

x	1	2	3	4	5
$f(x)$	9	4	0	–3	–5

87. The table above gives values of a function f at selected values of x. If f is twice-differentiable on the interval $1 \le x \le 5$, which of the following statements could be true?

(A) f' is negative and decreasing for $1 \le x \le 5$.

(B) f' is negative and increasing for $1 \le x \le 5$.

(C) f' is positive and decreasing for $1 \le x \le 5$.

(D) f' is positive and increasing for $1 \le x \le 5$.

GO ON TO THE NEXT PAGE.

88. Let f be the function defined by $f(x) = \ln(x^2 + 1)$, and let g be the function defined by $g(x) = x^5 + x^3$. The line tangent to the graph of f at $x = 2$ is parallel to the line tangent to the graph of g at $x = a$, where a is a positive constant. What is the value of a ?

(A) 0.246 (B) 0.430 (C) 0.447 (D) 0.790

GO ON TO THE NEXT PAGE.

89. Let f be a continuous function for all real numbers. Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$. If the average rate of change of g on the interval $2 \le x \le 5$ is 6, which of the following statements must be true?

(A) The average value of f on the interval $2 \le x \le 5$ is 6.

(B) $g'(2) = 6$

(C) $\dfrac{g'(5) + g'(2)}{2} = 6$

(D) $\int_2^5 g(x)\,dx = 6$

GO ON TO THE NEXT PAGE.

90. For any function f, which of the following statements must be true?

I. If f is defined at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

II. If f is continuous at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

III. If f is differentiable at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

(A) III only

(B) I and II only

(C) II and III only

(D) I, II, and III

GO ON TO THE NEXT PAGE.

END OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART B ONLY.**

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND ANSWER QUESTIONS 91–94.

Section II: Free-Response Questions

This is the free-response section of the 2017 AP exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP® Calculus AB Exam

SECTION II: Free Response

2017

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.33%

Part B

Number of Questions
4
Time
1 hour
Electronic Device
None allowed
Percent of Section II Score
66.67%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
   ☐☐☐☐☐☐
4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During Part B, you may continue to work on the questions in Part A without the use of a calculator.

As you begin each part, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 4NBP4-S

66

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

GO ON TO THE NEXT PAGE.

1. A particle moves along the x-axis so that its velocity at time t is given by $v(t) = \dfrac{t^6 - 13t^4 + 12}{10t^3 + 3}$. At time $t = 0$, the initial position of the particle is $x = 7$.

(a) Find the acceleration of the particle at time $t = 5.1$.

(b) Find all values of t in the interval $0 \le t \le 2$ for which the speed of the particle is 1.

Do not write beyond this border.

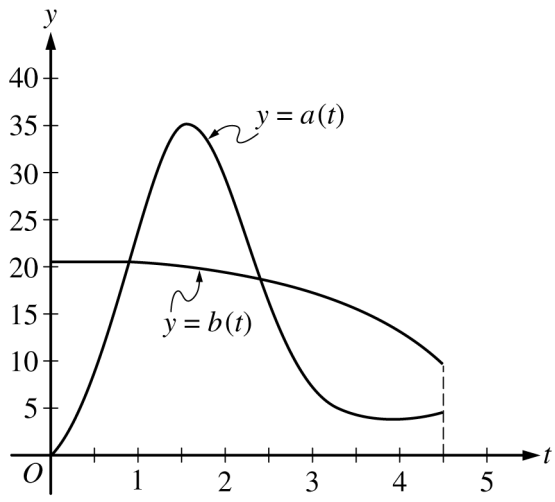
Continue question 1 on page 5.

(c) Find the position of the particle at time $t = 4$. Is the particle moving toward the origin or away from the origin at time $t = 4$? Justify your answer.

(d) During the time interval $0 < t \leq 4$, does the particle return to its initial position? Give a reason for your answer.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.



2. During the time interval $0 \le t \le 4.5$ hours, water flows into tank A at a rate of $a(t) = (2t - 5) + 5e^{2 \sin t}$ liters per hour. During the same time interval, water flows into tank B at a rate of $b(t)$ liters per hour. Both tanks are empty at time $t = 0$. The graphs of $y = a(t)$ and $y = b(t)$, shown in the figure above, intersect at $t = k$ and $t = 2.416$.

(a) How much water will be in tank A at time $t = 4.5$?

(b) During the time interval $0 \le t \le k$ hours, water flows into tank B at a constant rate of 20.5 liters per hour. What is the difference between the amount of water in tank A and the amount of water in tank B at time $t = k$?

Do not write beyond this border.

Continue question 2 on page 7.

(c) The area of the region bounded by the graphs of $y = a(t)$ and $y = b(t)$ for $k \le t \le 2.416$ is 14.470. How much water is in tank B at time $t = 2.416$?

(d) During the time interval $2.7 \le t \le 4.5$ hours, the rate at which water flows into tank B is modeled by $w(t) = 21 - \dfrac{30t}{(t-8)^2}$ liters per hour. Is the difference $w(t) - a(t)$ increasing or decreasing at time $t = 3.5$? Show the work that leads to your answer.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION II, Part B

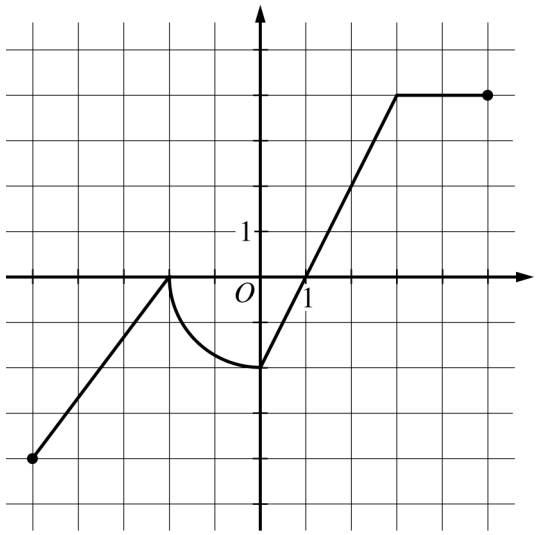
Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED



Graph of f

3. The graph of the function f, consisting of three line segments and a quarter of a circle, is shown above. Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the average rate of change of g from $x = -5$ to $x = 5$.

(b) Find the instantaneous rate of change of g with respect to x at $x = 3$, or state that it does not exist.

Do not write beyond this border.

Continue question 3 on page 15.

(c) On what open intervals, if any, is the graph of g concave up? Justify your answer.

(d) Find all x-values in the interval $-5 < x < 5$ at which g has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

Do not write beyond this border.

NO CALCULATOR ALLOWED

x	0	1	2	3	4	5	6
$f'(x)$	4	3.5	2	0.8	1.7	5.8	7

4. The function f satisfies $f(0) = 20$. The first derivative of f satisfies the inequality $0 \le f'(x) \le 7$ for all x in the closed interval $[0, 6]$. Selected values of f' are shown in the table above. The function f has a continuous second derivative for all real numbers.

   (a) Use a midpoint Riemann sum with three subintervals of equal length indicated by the data in the table to approximate the value of $f(6)$.

   (b) Determine whether the actual value of $f(6)$ could be 70. Explain your reasoning.

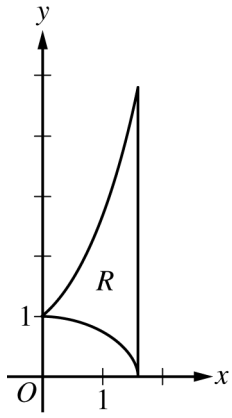
Do not write beyond this border.

Continue question 4 on page 17.

(c) Evaluate $\int_2^4 f''(x)\,dx$.

(d) Find $\lim_{x \to 0} \dfrac{f(x) - 20e^x}{0.5f(x) - 10}$.

NO CALCULATOR ALLOWED



5. Let R be the region in the first quadrant enclosed by the graph of $f(x) = \sqrt{\cos x}$, the graph of $g(x) = e^x$, and the vertical line $x = \dfrac{\pi}{2}$, as shown in the figure above.

   (a) Write, but do not evaluate, an integral expression that gives the area of R.

Do not write beyond this border.

Continue question 5 on page 19.

NO CALCULATOR ALLOWED

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Region R is the base of a solid whose cross sections perpendicular to the x-axis are semicircles with diameters on the xy-plane. Write, but do not evaluate, an integral expression that gives the volume of this solid.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

6. For $0 \le t \le 6$ seconds, a screen saver on a computer screen shows two circles that start as dots and expand outward.

(a) At the instant that the first circle has a radius of 9 centimeters, the radius is increasing at a rate of $\frac{3}{2}$ centimeters per second. Find the rate at which the area of the circle is changing at that instant. Indicate units of measure.

(b) The radius of the first circle is modeled by $w(t) = 12 - 12e^{-0.5t}$ for $0 \le t \le 6$, where $w(t)$ is measured in centimeters and t is measured in seconds. At what time t is the radius of the circle increasing at a rate of 3 centimeters per second?

Do not write beyond this border.

Continue question 6 on page 21.

NO CALCULATOR ALLOWED

(c) A model for the radius of the second circle is given by the function f for $0 \le t \le 6$, where $f(t)$ is measured in centimeters and t is measured in seconds. The rate of change of the radius of the second circle is given by $f'(t) = t^2 - 4t + 4$. Based on this model, by how many centimeters does the radius of the second circle increase from time $t = 0$ to $t = 3$?

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE FRONT COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**