From the 2018 Administration

AP® CollegeBoard

AP Calculus AB

Practice Exam

AP® Calculus AB Exam

SECTION I: Multiple Choice

2018

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
30
Time
1 hour
Electronic Device
None allowed

Part B

Number of Questions
15
Time
45 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 30 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 90 on page 3 of the answer sheet. Because Part A and Part B offer only four answer options for each question, do not mark the (E) answer circle for any question. The survey questions are numbers 91 through 94.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Form I
Form Code 40BP4-S

66

CALCULUS AB

SECTION I, Part A

Time—1 hour

Number of questions—30

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. If f is the function given by $f(x) = \dfrac{4}{x} + 5x - 1$, then $f'(2) =$

   (A) 4 (B) 6 (C) 7 (D) 11

---

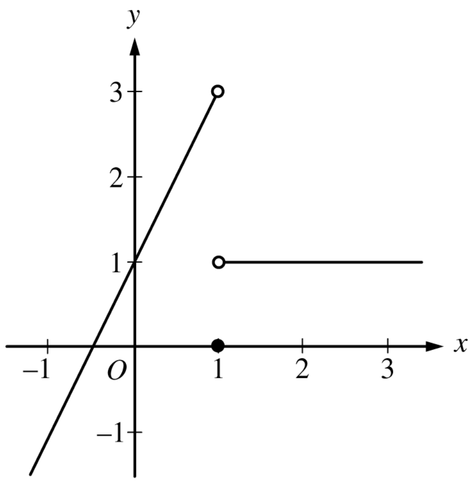
2. $\int 6e^{3x}\, dx =$

   (A) $2e^{3x} + C$

   (B) $6e^{3x} + C$

   (C) $18e^{3x} + C$

   (D) $\dfrac{6e^{3x+1}}{3x+1} + C$



Graph of f

3. The graph of $y = f(x)$ is shown above. What is $\lim_{x \to 1} f(x)$?

(A) 0 (B) 1 (C) 3 (D) The limit does not exist.

4. If $f'(x) = 3x^2 + 2x$ and $f(2) = 3$, then $f(1) =$

(A) -10 (B) -7 (C) 10 (D) 13

---

t (minutes)	0	5	10	15
$R(t)$ (people per minute)	100	100	75	55

5. During an evacuation drill, people leave a building at a rate of $R(t)$ people per minute, where t is the number of minutes since the start of the drill. Selected values of $R(t)$ are shown in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of people who leave the building during the first 15 minutes of the evacuation drill?

(A) 230 (B) 1150 (C) 1375 (D) 2075

6. If $y = x^2\left(e^x - 1\right)$, then $\dfrac{dy}{dx} =$

(A) $2xe^x$

(B) $2xe^x - 2x$

(C) $x^2e^x + 2xe^x - 2x$

(D) $x^2e^x + 2xe^x - x^2 - 2x$

---

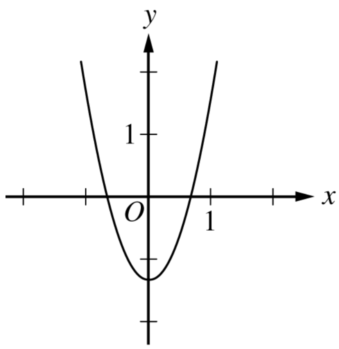
7. A particle moves along the x-axis so that at any time t, $t \geq 0$, its acceleration is $a(t) = -4\sin(2t)$. If the velocity of the particle at $t = 0$ is $v(0) = 7$ and its position at $t = 0$ is $x(0) = 0$, then its position at time t is $x(t) =$

(A) $\sin(2t) + 5t$

(B) $\sin(2t) + 7t$

(C) $\sin(2t) + 9t$

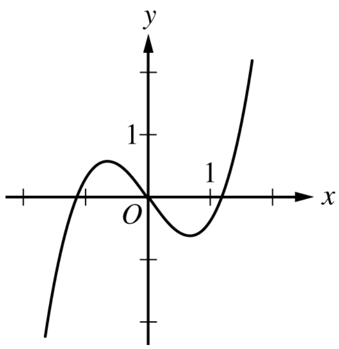
(D) $16\sin(2t) + 7t$



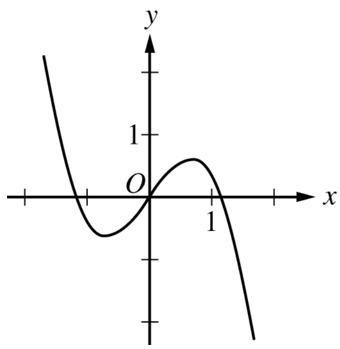
Graph of f''

8. The graph of f'', the second derivative of the function f, is shown above. Which of the following could be the graph of f ?

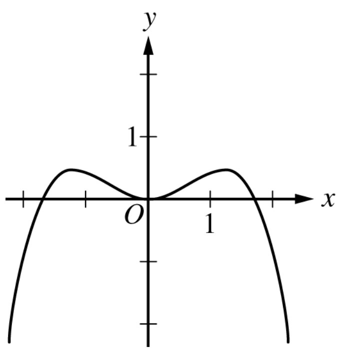
(A)



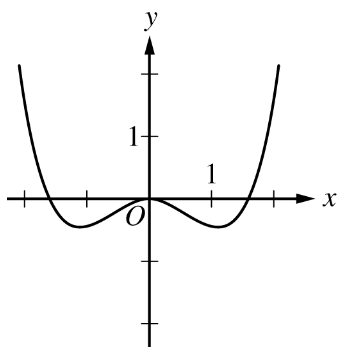
(B)



(C)



(D)



GO ON TO THE NEXT PAGE.

9. When $x = 2e$, $\lim\limits_{h \to 0} \dfrac{\ln(x+h) - \ln(x)}{h}$ is

(A) $\dfrac{1}{2e}$ (B) 1 (C) $\ln(2e)$ (D) nonexistent

---

10. If $\dfrac{dy}{dx} = x^4 - 2x^3 + 3x - 1$, then $\dfrac{d^3y}{dx^3}$ evaluated at $x = 2$ is

(A) 11 (B) 24 (C) 26 (D) 125

$$f(x) = \begin{cases} x^2 & \text{for } x < 0 \\ -1 & \text{for } x = 0 \\ x & \text{for } x > 0 \end{cases}$$

11. Let f be the function defined above. What is $\int_{-1}^{1} f(x)\,dx$?

(A) $\frac{5}{6}$ (B) $\frac{2}{3}$ (C) $-\frac{1}{6}$ (D) nonexistent

GO ON TO THE NEXT PAGE.

12. Given that $3x - \tan y = 4$, what is $\frac{dy}{dx}$ in terms of y ?

(A) $\frac{dy}{dx} = 3\sin^2 y$

(B) $\frac{dy}{dx} = 3\cos^2 y$

(C) $\frac{dy}{dx} = 3\cos y \cot y$

(D) $\frac{dy}{dx} = \frac{3}{1 + 9y^2}$

GO ON TO THE NEXT PAGE.

13. For time $t \geq 1$, the position of a particle moving along the x-axis is given by $p(t) = \sqrt{t} - 2$. At what time t in the interval $1 \leq t \leq 16$ is the instantaneous velocity of the particle equal to the average velocity of the particle over the interval $1 \leq t \leq 16$?

(A) 1 (B) $\frac{121}{25}$ (C) $\frac{25}{4}$ (D) 25

---

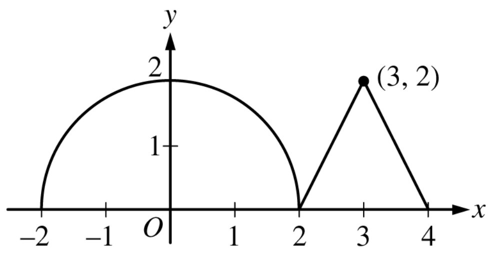
14. If f is a differentiable function and $y = \sin\left(f\left(x^2\right)\right)$, what is $\frac{dy}{dx}$ when $x = 3$?

(A) $\cos(f'(9))$

(B) $6\cos(f(9))$

(C) $f'(9)\cos(f(9))$

(D) $6f'(9)\cos(f(9))$



Graph of g'

15. The graph of g', the first derivative of the function g, consists of a semicircle of radius 2 and two line segments, as shown in the figure above. If $g(0) = 1$, what is $g(3)$?

(A) $\pi + 1$ (B) $\pi + 2$ (C) $2\pi + 1$ (D) $2\pi + 2$

---

16. Let f be the function given by $f(x) = x^3 - 6x^2 - 15x$. What is the maximum value of f on the interval $[0, 6]$?

(A) 0 (B) 5 (C) 6 (D) 8

17. $$\int \frac{1}{x^2 + 4x + 5}\, dx =$$

(A) $\arctan(x + 2) + C$

(B) $\arcsin(x + 2) + C$

(C) $\ln\left|x^2 + 4x + 5\right| + C$

(D) $\dfrac{1}{\frac{1}{3}x^3 + 2x^2 + 5x} + C$

---

18. Let f be the function defined by $f(x) = \sqrt[3]{x}$. What is the approximation for $f(10)$ found by using the line tangent to the graph of f at the point $(8, 2)$?

(A) $\frac{11}{6}$ (B) $\frac{25}{12}$ (C) $\frac{13}{6}$ (D) $\frac{7}{3}$

19. $\lim\limits_{x\to 0} \dfrac{4x^2}{e^{4x} - 4x - 1}$ is

(A) 0 (B) $\dfrac{1}{2}$ (C) 8 (D) nonexistent

---

20. Let g be a twice-differentiable, increasing function of t. If $g(0) = 20$ and $g(10) = 220$, which of the following must be true on the interval $0 < t < 10$?

(A) $g'(t) = 0$ for some t in the interval.

(B) $g'(t) = 20$ for some t in the interval.

(C) $g''(t) = 0$ for some t in the interval.

(D) $g''(t) > 0$ for all t in the interval.

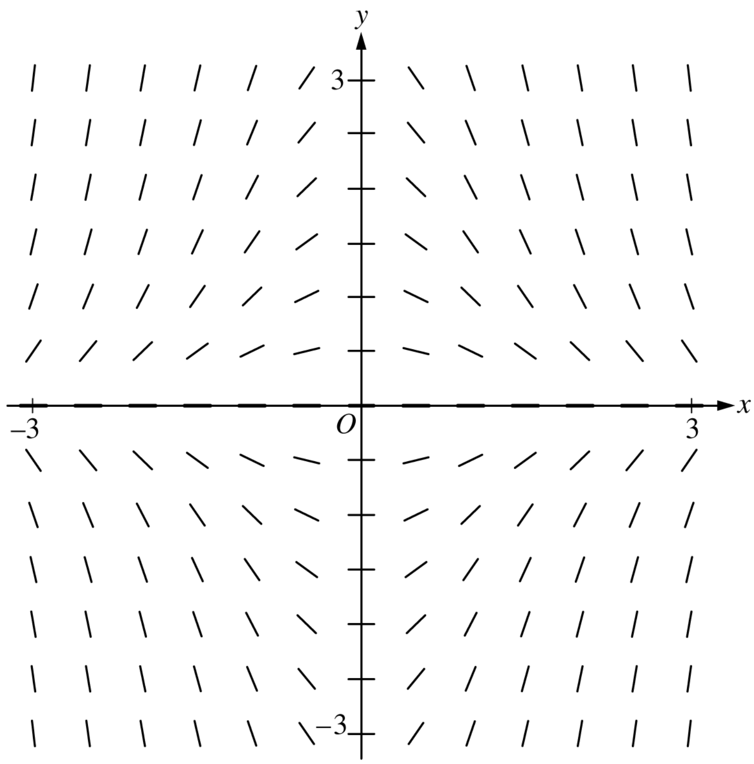
21. $\dfrac{d}{dx}\displaystyle\int_e^{x^3} \ln\left(t^2 + 1\right)\, dt =$

(A) $\ln\left(x^6 + 1\right)$

(B) $3x^2\ln\left(x^2 + 1\right)$

(C) $3x^2\ln\left(x^6 + 1\right)$

(D) $\ln\left(x^6 + 1\right) - \ln\left(e^2 + 1\right)$



22. Shown above is a slope field for which of the following differential equations?

(A) $\dfrac{dy}{dx} = \dfrac{x}{y}$

(B) $\dfrac{dy}{dx} = -\dfrac{x}{y}$

(C) $\dfrac{dy}{dx} = xy$

(D) $\dfrac{dy}{dx} = -xy$

GO ON TO THE NEXT PAGE.

23. Using the substitution $u = x + 1$, $\int \frac{x}{\sqrt{x+1}}\,dx$ is equivalent to

(A) $\int \frac{1}{u+1}\,du$ (B) $\int u^{-1/2}\,du$ (C) $\int \left(u^{1/2} - u^{-1/2}\right) du$ (D) $(u-1)\int u^{-1/2}\,du$

---

24. Let f be the function given by $f(x) = \frac{2x^2 + 14x - 16}{x^2 - 9x + 8}$. For what values of x does f have a removable discontinuity?

(A) 1 only (B) 8 only (C) -8 and 1 (D) 1 and 8

25. Which of the following is a solution to the differential equation $y'' - 4y = 0$?

(A) $y = e^{2x}$ (B) $y = 2e^{x}$ (C) $y = \sin(2x)$ (D) $y = \cos(2x)$

GO ON TO THE NEXT PAGE.

x	10	11	12	13	14
$f(x)$	5	2	3	6	5

26. The table above gives values of the continuous function f at selected values of x. If f has exactly two critical points on the open interval $(10, 14)$, which of the following must be true?

(A) $f(x) > 0$ for all x in the open interval $(10, 14)$.

(B) $f'(x)$ exists for all x in the open interval $(10, 14)$.

(C) $f'(x) < 0$ for all x in the open interval $(10, 11)$.

(D) $f'(12) \neq 0$

27. The positive variables p and c change with respect to time t. The relationship between p and c is given by the equation $p^2 = (20 - c)^3$. At the instant when $\frac{dp}{dt} = 41$ and $c = 15$, what is the value of $\frac{dc}{dt}$?

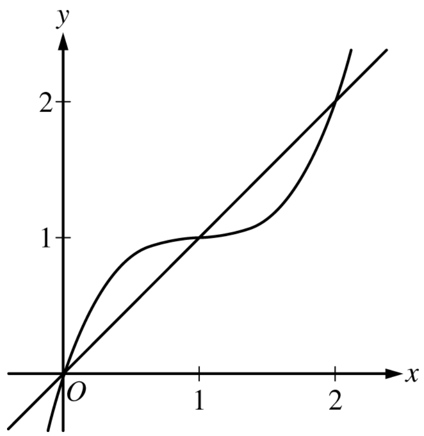
(A) $-\frac{82}{75}$ (B) $-\frac{2\sqrt{5}}{3}$ (C) $-\frac{3\sqrt{5}}{2}$ (D) $-\frac{82\sqrt{5}}{15}$

---

28. $\lim\limits_{x \to -\infty} \frac{3 + 2^x}{4 - 5^x}$ is

(A) $-\frac{2}{5}$ (B) 0 (C) $\frac{3}{4}$ (D) nonexistent

GO ON TO THE NEXT PAGE.



29. The graphs of the function g and the line $y = x$ are shown in the figure above. The graphs intersect at the points $(0, 0)$, $(1, 1)$, and $(2, 2)$. Which of the following expressions give the area enclosed by the graphs?

I. $\left| \int_0^2 (x - g(x))\, dx \right|$

II. $\int_0^2 \left| x - g(x) \right| dx$

III. $\int_0^1 (g(x) - x)\, dx + \int_1^2 (x - g(x))\, dx$

(A) II only

(B) III only

(C) I and II only

(D) II and III only

30. A student attempts to solve the differential equation $\frac{dy}{dx} = xy^3$ with the initial condition that $y = 2$ when $x = 0$. The steps of the student's solution are shown below. In which of the following steps does an error first appear?

Step 1: $\int \frac{1}{y^3}\, dy = \int x\, dx$

Step 2: $\ln\left|y^3\right| = \frac{x^2}{2} + C$

Step 3: $\left|y^3\right| = Ke^{x^2/2}$

Step 4: $\left|y^3\right| = 4e^{x^2/2}$

Step 5: $y = \sqrt[3]{4e^{x^2/2}}$

(A) Step 1 (B) Step 2 (C) Step 3 (D) Step 4

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

PART B STARTS ON PAGE 26.

CALCULUS AB

SECTION I, Part B

Time—45 minutes

Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–90.

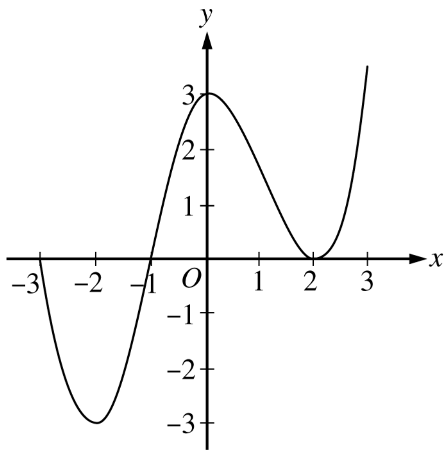
YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f'

76. The graph of f', the derivative of the function f, is shown above for $-3 \le x \le 3$. On what intervals is f increasing?

(A) $[-3, -1]$ only (B) $[-1, 3]$ (C) $[-2, 0]$ and $[2, 3]$ (D) $[-3, -1]$ and $[1, 3]$

GO ON TO THE NEXT PAGE.

77. The rate at which water leaks from a tank, in gallons per hour, is modeled by R, a differentiable function of the number of hours after the leak is discovered. Which of the following is the best interpretation of $R'(3)$?

(A) The amount of water, in gallons, that has leaked out of the tank during the first three hours after the leak is discovered

(B) The amount of change, in gallons per hour, in the rate at which water is leaking during the three hours after the leak is discovered

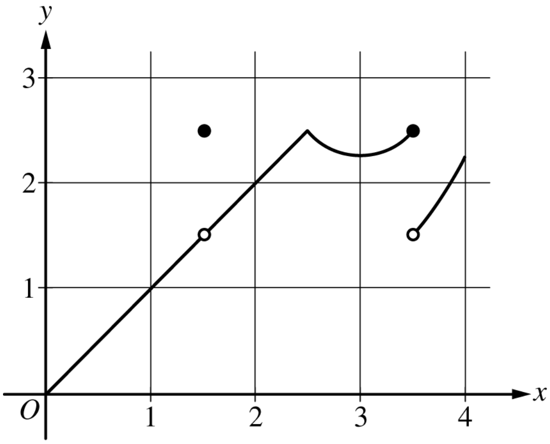
(C) The rate at which water leaks from the tank, in gallons per hour, three hours after the leak is discovered

(D) The rate of change of the rate at which water leaks from the tank, in gallons per hour per hour, three hours after the leak is discovered

---

78. A particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \frac{4}{t^3 + 1}$. If the position of the particle is $x = 1$ when $t = 2$, what is the position of the particle when $t = 4$?

(A) 0.617 (B) 0.647 (C) 1.353 (D) 5.713



Graph of f

79. The graph of the function f is shown above. Of the following intervals, on which is f continuous but not differentiable?

(A) $(0, 1)$ (B) $(1, 2)$ (C) $(2, 3)$ (D) $(3, 4)$

GO ON TO THE NEXT PAGE.

80. The first derivative of the function f is defined by $f'(x) = \left(x^2 + 1\right)\sin(3x - 1)$ for $-1.5 < x < 1.5$. On which of the following intervals is the graph of f concave up?

(A) $(-1.5, -1.341)$ and $(-0.240, 0.964)$

(B) $(-1.341, -0.240)$ and $(0.964, 1.5)$

(C) $(-0.714, 0.333)$ and $(1.381, 1.5)$

(D) $(-1.5, -0.714)$ and $(0.333, 1.381)$

---

81. During a rainfall, the depth of water in a rain gauge increases at a rate modeled by $R(t) = 0.5 + t\cos\left(\frac{\pi t^3}{80}\right)$, where t is the time in hours since the start of the rainfall and $R(t)$ is measured in centimeters per hour. How much did the depth of water in the rain gauge increase from $t = 0$ to $t = 3$ hours?

(A) 1.233 cm (B) 1.466 cm (C) 1.966 cm (D) 5.401 cm

82. Let f be a function such that $f(1) = -2$ and $f(5) = 7$. Which of the following conditions ensures that $f(c) = 0$ for some value c in the open interval $(1, 5)$?

(A) $\int_1^5 f(x)\,dx$ exists.

(B) f is increasing on the closed interval [1, 5].

(C) f is continuous on the closed interval [1, 5].

(D) f is defined for all values of x in the closed interval [1, 5].

---

83. The acceleration of a particle moving along the x-axis is given by $a(t) = (t - 8)\sin t$ for $0 \le t \le 8$. At what value of t is the particle's velocity decreasing most rapidly?

(A) 0 (B) 1.420 (C) 3.142 (D) 4.439

84. If the average value of the function f over the closed interval [2, 4] is 3 and if $f(x) \geq 0$ for all x in [2, 4], what is the area of the region enclosed by the graph of $y = f(x)$, the lines $x = 2$ and $x = 4$, and the x-axis?

(A) 12 (B) 6 (C) 3 (D) $\frac{3}{2}$

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$	–0.1054	–0.0101	–0.001	0.001	0.0099	0.0953

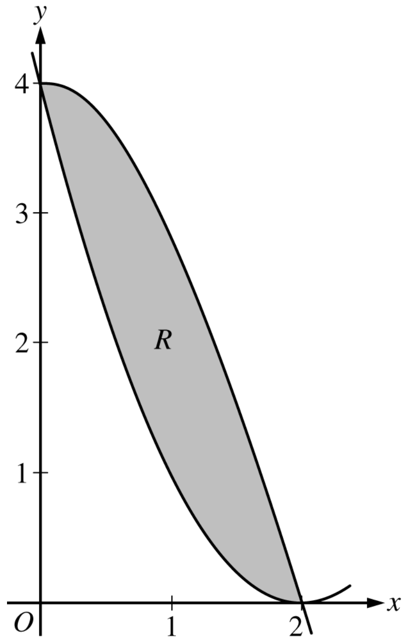
85. The function f is continuous and increasing for $x > -1$. The table above gives values of f at selected values of x. Of the following, which is the best approximation for $\lim_{x \to 0} e^{-2f(x)}$?

(A) -2

(B) 0

(C) 1

(D) The limit does not exist.



86. Let R be the region in the first quadrant bounded by the graphs of $y = 4\cos\left(\frac{\pi x}{4}\right)$ and $y = (x-2)^2$, as shown in the figure above. The region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in region R. What is the volume of the solid?

(A) 1.775 (B) 3.549 (C) 4.800 (D) 5.575

87. Let f and g be continuous functions. If $\int_2^6 f(x)\,dx = 5$ and $\int_6^2 g(x)\,dx = 7$, then $\int_2^6 (3f(x) + g(x))\,dx =$

(A) -6 (B) 8 (C) 22 (D) 36

88. Let f be a twice-differentiable function such that $f''(x) < 0$ for all x. The graph of $y = S(x)$ is the secant line passing through the points $(3, f(3))$ and $(5, f(5))$. The graph of $y = T(x)$ is the line tangent to the graph of f at $x = 4$. Which of the following is true?

(A) $f(4.2) < S(4.2) < T(4.2)$

(B) $f(4.2) < T(4.2) < S(4.2)$

(C) $S(4.2) < f(4.2) < T(4.2)$

(D) $T(4.2) < f(4.2) < S(4.2)$

89. The number of insects in a certain population at time t days is modeled by the function P with first derivative $P'(t) = 0.3t^2 + 12t + 210$. At time $t = 0$, the number of insects in the population is 40. Which of the following statements are true?

I. At time $t = 10$, the number of insects in the population is 2840.

II. At time $t = 10$, the number of insects in the population is increasing at a rate of 360 insects per day.

III. At time $t = 10$, the rate of change of the number of insects in the population is increasing at a rate of 18 insects per day per day.

(A) I only (B) II only (C) III only (D) I, II, and III

x	3	7
$h(x)$	7	22
$h'(x)$	5	10

90. Selected values of the increasing function h and its derivative h' are shown in the table above. If g is a differentiable function such that $h(g(x)) = x$ for all x, what is the value of $g'(7)$?

(A) $-\frac{1}{10}$ (B) $\frac{1}{10}$ (C) $\frac{1}{5}$ (D) $\frac{7}{5}$

GO ON TO THE NEXT PAGE.

END OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART B ONLY.**

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND ANSWER QUESTIONS 91–94.

Section II: Free-Response Questions

This is the free-response section of the 2018 AP Exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP® Calculus AB Exam

SECTION II: Free Response

2018

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.33%

Part B

Number of Questions
4
Time
1 hour
Electronic Device
None allowed
Percent of Section II Score
66.67%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ ☐☐ ☐☐☐☐
   Month Day Year
3. Six-digit school code
   ☐☐☐☐☐☐
4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark “No” with no effect on my score or its reporting.
   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During Part B, you may continue to work on the questions in Part A without the use of a calculator.

As you begin each part, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 40BP4-S

66

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (minutes)	0	1	5	6	8
$g(t)$ (cubic feet per minute)	12.8	15.1	20.5	18.3	22.7

1. Grain is being added to a silo. At time $t = 0$, the silo is empty. The rate at which grain is being added is modeled by the differentiable function g, where $g(t)$ is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Selected values of $g(t)$ are given in the table above.

(a) Using the data in the table, approximate $g'(3)$. Using correct units, interpret the meaning of $g'(3)$ in the context of the problem.

(b) Write an integral expression that represents the total amount of grain added to the silo from time $t = 0$ to time $t = 8$. Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate the integral.

Do not write beyond this border.

Continue question 1 on page 5.

(c) The grain in the silo is spoiling at a rate modeled by $w(t) = 32 \cdot \sqrt{\sin\left(\frac{\pi t}{74}\right)}$, where $w(t)$ is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Using the result from part (b), approximate the amount of unspoiled grain remaining in the silo at time $t = 8$.

(d) Based on the model in part (c), is the amount of unspoiled grain in the silo increasing or decreasing at time $t = 6$? Show the work that leads to your answer.

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

2. A snail is traveling along a straight path. The snail's velocity can be modeled by $v(t) = 1.4 \ln\left(1 + t^2\right)$ inches per minute for $0 \le t \le 15$ minutes.

(a) Find the acceleration of the snail at time $t = 5$ minutes.

(b) What is the displacement of the snail over the interval $0 \le t \le 15$ minutes?

Do not write beyond this border.

Continue question 2 on page 7.

(c) At what time t, $0 \le t \le 15$, is the snail's instantaneous velocity equal to its average velocity over the interval $0 \le t \le 15$?

(d) An ant arrives at the snail's starting position at time $t = 12$ minutes and follows the snail's path. During the interval $12 \le t \le 15$ minutes, the ant travels in the same direction as the snail with a constant acceleration of 2 inches per minute per minute. The ant catches up to the snail at time $t = 15$ minutes. The ant's velocity at time $t = 12$ is B inches per minute. Find the value of B.

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION II, Part B

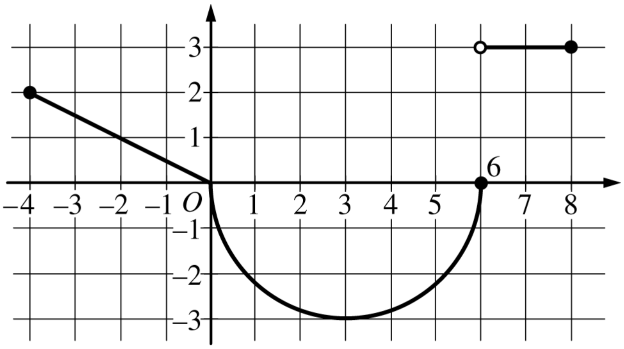
Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED



Graph of g

3. The function g is defined on the closed interval $[-4, 8]$. The graph of g consists of two linear pieces and a semicircle, as shown in the figure above. Let f be the function defined by $f(x) = 3x + \int_0^x g(t)\, dt$.

(a) Find $f(7)$ and $f'(7)$.

(b) Find the value of x in the closed interval $[-4, 3]$ at which f attains its maximum value. Justify your answer.

Continue question 3 on page 15.

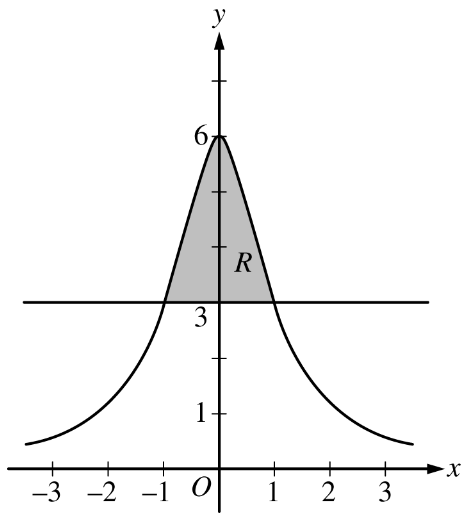
Do not write beyond this border.

NO CALCULATOR ALLOWED

(c) For each of $\lim_{x \to 0^-} g'(x)$ and $\lim_{x \to 0^+} g'(x)$, find the value or state that it does not exist.

(d) Find $\lim_{x \to -2} \dfrac{f(x) + 7}{e^{3x+6} - 1}$.

NO CALCULATOR ALLOWED



4. Let f be the function defined by $f(x) = \dfrac{6}{1 + x^2}$. Let R be the shaded region bounded by the graph of f and the horizontal line $y = 3$, as shown in the figure above.

(a) Find the area of R.

Continue question 4 on page 17.

NO CALCULATOR ALLOWED

(b) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Let $h(x)$ be the vertical distance between the point $(x, f(x))$ and the horizontal line $y = 3$. Find the rate of change of $h(x)$ with respect to x at $x = 2$.

Do not write beyond this border.

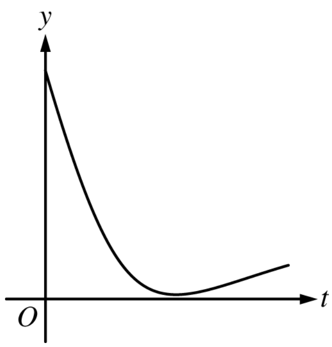
Do not write beyond this border.

NO CALCULATOR ALLOWED

5. During a chemical reaction, the function $y = f(t)$ models the amount of a substance present, in grams, at time t seconds. At the start of the reaction $(t = 0)$, there are 10 grams of the substance present. The function $y = f(t)$ satisfies the differential equation $\frac{dy}{dt} = -0.02y^2$.

(a) Use the line tangent to the graph of $y = f(t)$ at $t = 0$ to approximate the amount of the substance remaining at time $t = 2$ seconds.

(b) Using the given differential equation, determine whether the graph of f could resemble the following graph. Give a reason for your answer.



Do not write beyond this border.

Continue question 5 on page 19.

NO CALCULATOR ALLOWED

(c) Find an expression for $y = f(t)$ by solving the differential equation $\frac{dy}{dt} = -0.02y^2$ with the initial condition $f(0) = 10$.

(d) Determine whether the amount of the substance is changing at an increasing or a decreasing rate. Explain your reasoning.

Do not write beyond this border.

Do not write beyond this border.

6. Consider the curve given by the equation $2(x - y) = 3 + \cos y$. For all points on the curve, $\frac{2}{3} \le \frac{dy}{dx} \le 2$.

(a) Show that $\frac{dy}{dx} = \frac{2}{2 - \sin y}$.

(b) For $-\frac{\pi}{2} < y < \frac{\pi}{2}$, there is a point P on the curve through which the line tangent to the curve has slope 1.

Find the coordinates of the point P.

Continue question 6 on page 21.

NO CALCULATOR ALLOWED

(c) Determine the concavity of the curve at points for which $-\frac{\pi}{2} < y < \frac{\pi}{2}$. Give a reason for your answer.

(d) Let $y = f(x)$ be a function, defined implicitly by $2(x - y) = 3 + \cos y$, that is continuous on the closed interval $[2, 2.1]$ and differentiable on the open interval $(2, 2.1)$. Use the Mean Value Theorem on the interval $[2, 2.1]$ to show that $\frac{1}{15} \le f(2.1) - f(2) \le \frac{1}{5}$.

STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE FRONT COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**