From the 2017 Administration

AP® CollegeBoard

Calculus BC

Practice Exam

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AP® Calculus BC Exam

SECTION I: Multiple Choice

2017

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
30
Time
1 hour
Electronic Device
None allowed

Part B

Number of Questions
15
Time
45 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 30 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 90 on page 3 of the answer sheet. Because Part A and Part B offer only four answer options for each question, do not mark the (E) answer circle for any question. The survey questions are numbers 91 through 94.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Form I
Form Code 4NBP4-S

68

CALCULUS BC

SECTION I, Part A

Time—1 hour

Number of questions—30

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. If $f(x) = \cos^2(3x - 5)$, then $f'(x) =$

(A) $6\cos(3x - 5)$

(B) $-3\sin^2(3x - 5)$

(C) $-2\sin(3x - 5)\cos(3x - 5)$

(D) $-6\sin(3x - 5)\cos(3x - 5)$

---

2. $\displaystyle\int \frac{1}{t\sqrt{t}}\,dt =$

(A) $-2t^{-1/2} + C$

(B) $-\frac{3}{2}t^{-5/2} + C$

(C) $-\frac{2}{5}t^{-5/2} + C$

(D) $2t^{1/2}\ln t + C$

3. If $f(x) = \dfrac{5 - x}{x^3 + 2}$, then $f'(x) =$

(A) $\dfrac{-4x^3 + 15x^2 - 2}{\left(x^3 + 2\right)^2}$

(B) $\dfrac{-2x^3 + 15x^2 + 2}{\left(x^3 + 2\right)^2}$

(C) $\dfrac{2x^3 - 15x^2 - 2}{\left(x^3 + 2\right)^2}$

(D) $\dfrac{4x^3 - 15x^2 + 2}{\left(x^3 + 2\right)^2}$

4. The position of a particle moving in the xy-plane is given by the vector $\left\langle 4t^3, y(2t) \right\rangle$, where y is a twice-differentiable function of t. At time $t = \dfrac{1}{2}$, what is the acceleration vector of the particle?

(A) $\langle 3, 2y''(1) \rangle$

(B) $\langle 6, 4y''(1) \rangle$

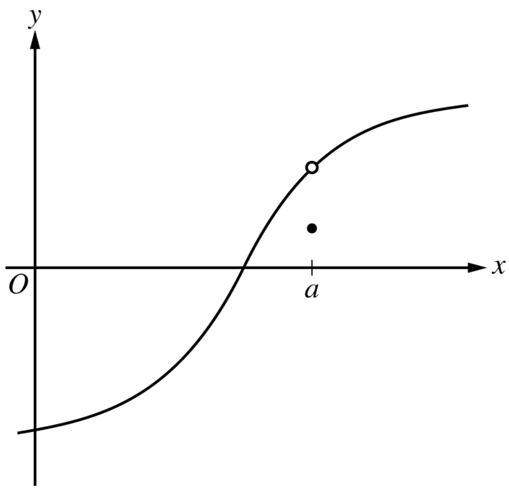
(C) $\langle 12, 2y''(1) \rangle$

(D) $\langle 12, 4y''(1) \rangle$

---

5. To what number does the series $\displaystyle\sum_{k=0}^{\infty} \left(\frac{-e}{\pi} \right)^k$ converge?

(A) 0 (B) $\dfrac{-e}{\pi + e}$ (C) $\dfrac{\pi}{\pi + e}$ (D) The series does not converge.



Graph of f

6. The graph of $y = f(x)$ is shown above. Which of the following is true?

(A) $\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$ exists.

(B) $\lim\limits_{x \to a^+} f(x) \neq \lim\limits_{x \to a^-} f(x)$

(C) $\lim\limits_{x \to a} f(x) \neq f(a)$

(D) $\lim\limits_{x \to a} f(x)$ does not exist.

7. If $\int_{4}^{-10} g(x)\,dx = -3$ and $\int_{4}^{6} g(x)\,dx = 5$, then $\int_{-10}^{6} g(x)\,dx =$

(A) -8 (B) -2 (C) 2 (D) 8

8. The length of the curve $y = \sin(3x)$ from $x = 0$ to $x = \dfrac{\pi}{6}$ is given by

(A) $\int_{0}^{\pi/6} \left(1 + 9\cos^2(3x)\right) dx$

(B) $\int_{0}^{\pi/6} \sqrt{1 + \sin^2(3x)}\, dx$

(C) $\int_{0}^{\pi/6} \sqrt{1 + 3\cos(3x)}\, dx$

(D) $\int_{0}^{\pi/6} \sqrt{1 + 9\cos^2(3x)}\, dx$

GO ON TO THE NEXT PAGE.

9. The slope of the line tangent to the graph of $y = xe^x$ at $x = \ln 2$ is

(A) $2 \ln 2$ (B) $2 \ln 2 + 2$ (C) $e^2(\ln 2) + e^2$ (D) $2 + \dfrac{2 \ln 2}{e}$

---

10. Let $y = f(x)$ be the solution to the differential equation $\dfrac{dy}{dx} = x - y$ with initial condition $f(2) = 8$. What is the approximation for $f(3)$ obtained by using Euler's method with two steps of equal length, starting at $x = 2$?

(A) 2 (B) $\dfrac{5}{2}$ (C) $\dfrac{15}{4}$ (D) $\dfrac{61}{4}$

11. If $x^2 + xy - 3y = 3$, then at the point $(2, 1)$, $\dfrac{dy}{dx} =$

(A) 5 (B) 4 (C) $\dfrac{7}{3}$ (D) 2

---

12. $$\int \frac{3x+1}{x^2 - 4x + 3}\,dx =$$

(A) $-2\ln|x-3| + 5\ln|x-1| + C$

(B) $\dfrac{1}{5}\ln|x-3| - \dfrac{1}{2}\ln|x-1| + C$

(C) $\dfrac{1}{2}\ln|x-3| - \dfrac{1}{2}\ln|x-1| + C$

(D) $5\ln|x-3| - 2\ln|x-1| + C$

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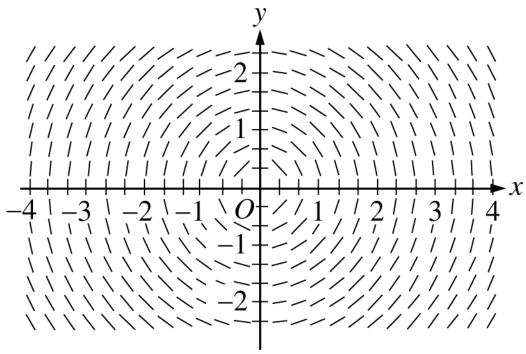
13. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = x^2 + y^2$?

(A)



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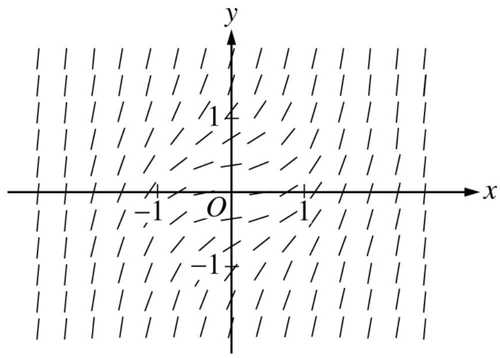
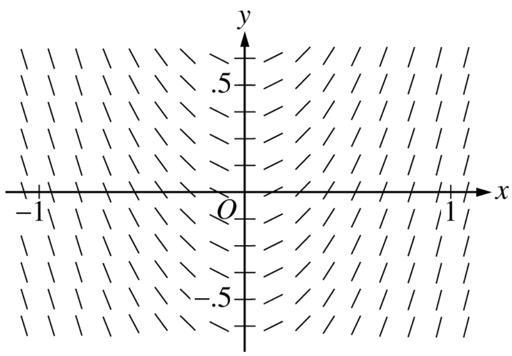


(C)



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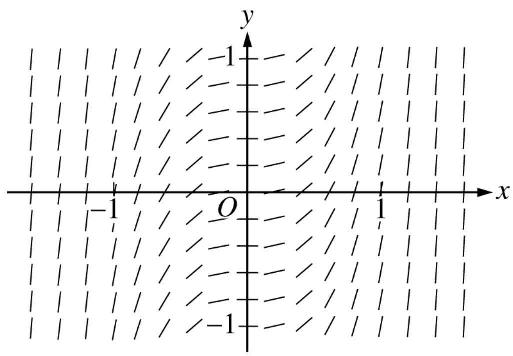

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14. If $f(x) = 3x^2 + 2x$, then $f'(x) =$

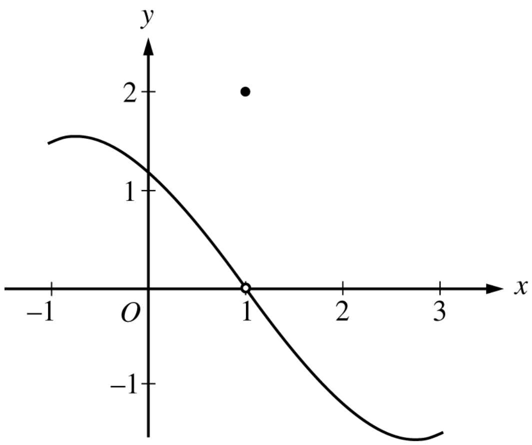
(A) $\displaystyle \lim_{h \to 0} \frac{\left(3x^2 + 2x + h\right) - \left(3x^2 + 2x\right)}{h}$

(B) $\displaystyle \lim_{x \to 0} \frac{\left(3x^2 + 2x + h\right) - \left(3x^2 + 2x\right)}{h}$

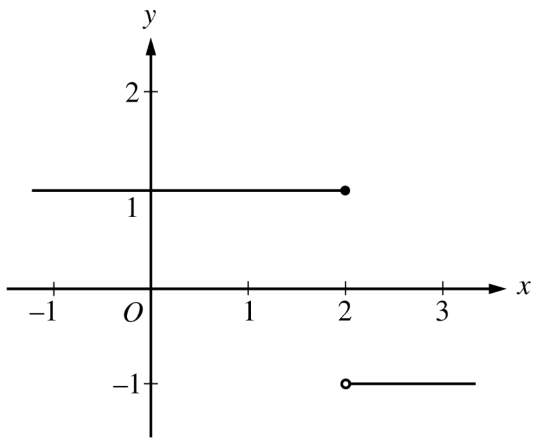
(C) $\displaystyle \lim_{h \to 0} \frac{\left(3(x + h)^2 + 2(x + h)\right) - \left(3x^2 + 2x\right)}{h}$

(D) $\displaystyle \lim_{x \to 0} \frac{\left(3(x + h)^2 + 2(x + h)\right) - \left(3x^2 + 2x\right)}{h}$

GO ON TO THE NEXT PAGE.



Graph of f



Graph of g

15. The graphs of the functions f and g are shown in the figures above. Which of the following statements is false?

(A) $\lim_{x \to 1} f(x) = 0$

(B) $\lim_{x \to 2} g(x)$ does not exist.

(C) $\lim_{x \to 1} \left(f(x)\, g(x+1) \right)$ does not exist.

(D) $\lim_{x \to 1} \left(f(x+1)\, g(x) \right)$ exists.

GO ON TO THE NEXT PAGE.

16. Which of the following is the interval of convergence for the series $\sum_{n=0}^{\infty} \frac{(x+2)^n}{2^n}$?

(A) $-4 < x < 0$

(B) $-4 \le x < 0$

(C) $-2 < x < 2$

(D) $-2 \le x < 2$

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17. $\int_0^5 \sqrt{\frac{5-x}{5}}\, dx =$

(A) $\frac{2}{3}$ (B) $\frac{10}{3}$ (C) 5 (D) $\frac{50\sqrt{5}}{3}$

GO ON TO THE NEXT PAGE.

18. Which of the following limits are equal to -1 ?

I. $\lim\limits_{x \to 0^-} \dfrac{|x|}{x}$

II. $\lim\limits_{x \to 3} \dfrac{x^2 - 7x + 12}{3 - x}$

III. $\lim\limits_{x \to \infty} \dfrac{1 - x}{1 + x}$

(A) I only (B) I and III only (C) II and III only (D) I, II, and III

---

19. Let f be the function given by $f(x) = 2\cos x + 1$. What is the approximation for $f(1.5)$ found by using the line tangent to the graph of f at $x = \dfrac{\pi}{2}$?

(A) -2 (B) 1 (C) $\pi - 2$ (D) $4 - \pi$

20. A particle moves in the xy-plane so that its position for $t \geq 0$ is given by the parametric equations $x = \ln(t + 1)$ and $y = kt^2$, where k is a positive constant. The line tangent to the particle's path at the point where $t = 3$ has slope 8. What is the value of k ?

(A) $\frac{1}{192}$ (B) $\frac{1}{3}$ (C) $\frac{4}{3}$ (D) $\frac{16}{3}$

Time (weeks)	0	2	6	10
Level	210	200	190	180

21. The table above gives the level of a person's cholesterol at different times during a 10-week treatment period. What is the average level over this 10-week period obtained by using a trapezoidal approximation with the subintervals [0, 2], [2, 6], and [6, 10] ?

(A) 188 (B) 193 (C) 195 (D) 198

22. $\int \frac{x}{2} e^{-3x/4}\, dx =$

(A) $-\frac{3x}{4} e^{-3x/4} + \frac{3}{4} e^{-3x/4} + C$

(B) $-\frac{2x}{3} e^{-3x/4} - \frac{8}{9} e^{-3x/4} + C$

(C) $-\frac{x}{2} e^{-3x/4} + \frac{3}{8} e^{-3x/4} + C$

(D) $\frac{x}{2} e^{-3x/4} - \frac{1}{2} e^{-3x/4} + C$

GO ON TO THE NEXT PAGE.

23. If $f(x) = \sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$, then $f'(x) =$

(A) $\frac{x^3}{3} + \frac{x^5}{5 \cdot 2!} + \frac{x^7}{7 \cdot 3!} + \frac{x^9}{9 \cdot 4!} + \cdots + \frac{x^{(2n+1)}}{(2n+1)n!} + \cdots$

(B) $x + \frac{3x^3}{2!} + \frac{5x^5}{3!} + \frac{7x^7}{4!} + \cdots + \frac{(2n-1)x^{(2n-1)}}{n!} + \cdots$

(C) $2 + 2x^2 + x^4 + \frac{x^6}{3} + \cdots + \frac{2x^{2(n-1)}}{(n-1)!} + \cdots$

(D) $2x + 2x^3 + x^5 + \frac{x^7}{3} + \cdots + \frac{2nx^{(2n-1)}}{n!} + \cdots$

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24. If the average value of a continuous function f on the interval $[-2, 4]$ is 12, what is $\int_{-2}^{4} \frac{f(x)}{8}\, dx$?

(A) $\frac{3}{2}$ (B) 3 (C) 9 (D) 72

25. What is the radius of convergence of the Maclaurin series for $\dfrac{2x}{1+x^2}$?

(A) $\dfrac{1}{2}$ (B) 1 (C) 2 (D) infinite

26. Let f be the function with $f(0) = \dfrac{1}{\pi^2}$, $f(2) = \dfrac{1}{\pi^2}$, and derivative given by $f'(x) = (x+1)\cos(\pi x)$. How many values of x in the open interval $(0, 2)$ satisfy the conclusion of the Mean Value Theorem for the function f on the closed interval $[0, 2]$?

(A) None
(B) One
(C) Two
(D) More than two

27. The number of students in a cafeteria is modeled by the function P that satisfies the logistic differential equation $\frac{dP}{dt} = \frac{1}{2000}P(200 - P)$, where t is the time in seconds and $P(0) = 25$. What is the greatest rate of change, in students per second, of the number of students in the cafeteria?

(A) 5 (B) 25 (C) 100 (D) 200

GO ON TO THE NEXT PAGE.

28. A cube with edges of length x centimeters has volume $V(x) = x^3$ cubic centimeters. The volume is increasing at a constant rate of 40 cubic centimeters per minute. At the instant when $x = 2$, what is the rate of change of x, in centimeters per minute, with respect to time?

(A) $\frac{10}{3}$ (B) $\sqrt{\frac{40}{3}}$ (C) 5 (D) 10

GO ON TO THE NEXT PAGE.

29. Which of the following is a power series expansion of $\dfrac{e^x + e^{-x}}{2}$?

(A) $1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \dfrac{x^6}{6!} + \cdots + \dfrac{x^{2n}}{(2n)!} + \cdots$

(B) $1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots + (-1)^n \dfrac{x^{2n}}{(2n)!} + \cdots$

(C) $x + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \dfrac{x^7}{7!} + \cdots + \dfrac{x^{2n+1}}{(2n+1)!} + \cdots$

(D) $x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots + (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} + \cdots$

GO ON TO THE NEXT PAGE.

30. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{1}{2^n - n}$ is true?

(A) The series diverges by the nth term test.

(B) The series diverges by limit comparison to the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$.

(C) The series converges by the nth term test.

(D) The series converges by limit comparison to the geometric series $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

PART B STARTS ON PAGE 26.

CALCULUS BC

SECTION I, Part B

Time—45 minutes

Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–90.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

76. Let f be a twice-differentiable function for all real numbers x. Which of the following additional properties guarantees that f has a relative minimum at $x = c$?

(A) $f'(c) = 0$

(B) $f'(c) = 0$ and $f''(c) < 0$

(C) $f'(c) = 0$ and $f''(c) > 0$

(D) $f'(x) > 0$ for $x < c$ and $f'(x) < 0$ for $x > c$

---

77. Let $H(x)$ be an antiderivative of $\dfrac{x^3 + \sin x}{x^2 + 2}$. If $H(5) = \pi$, then $H(2) =$

(A) -9.008 (B) -5.867 (C) 4.626 (D) 12.150

GO ON TO THE NEXT PAGE.

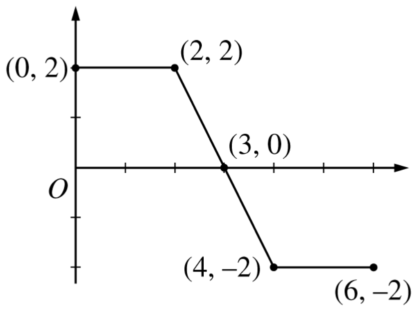
78. The continuous function f is positive and has domain $x > 0$. If the asymptotes of the graph of f are $x = 0$ and $y = 2$, which of the following statements must be true?

(A) $\lim\limits_{x \to 0^+} f(x) = \infty$ and $\lim\limits_{x \to 2} f(x) = \infty$

(B) $\lim\limits_{x \to 0^+} f(x) = 2$ and $\lim\limits_{x \to \infty} f(x) = 0$

(C) $\lim\limits_{x \to 0^+} f(x) = \infty$ and $\lim\limits_{x \to \infty} f(x) = 2$

(D) $\lim\limits_{x \to 2} f(x) = \infty$ and $\lim\limits_{x \to \infty} f(x) = 2$



Graph of f

79. The graph of a function f, consisting of three line segments, is shown above. The function f is defined on the closed interval [0, 6]. Let $g(x) = \int_2^x f(t)\,dt$. What is the maximum value of $g(x)$ for $0 \le x \le 6$?

(A) 0 (B) 1 (C) 5 (D) 10

80. The position of an object moving along a path in the xy-plane is given by the parametric equations $x(t) = 5\sin(\pi t)$ and $y(t) = (2t - 1)^2$. The speed of the particle at time $t = 0$ is

(A) 3.422

(B) 11.708

(C) 15.580

(D) 16.209

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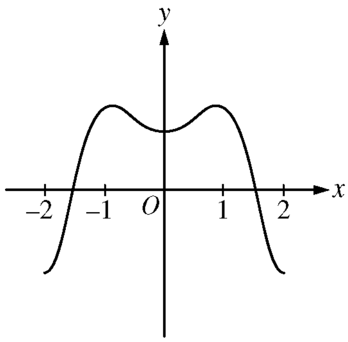
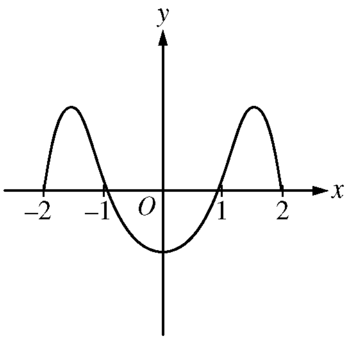


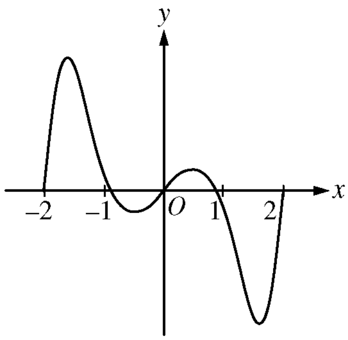
Graph of f

81. The graph of the function f is shown above for $-2 \le x \le 2$. Which of the following could be the graph of an antiderivative of f ?

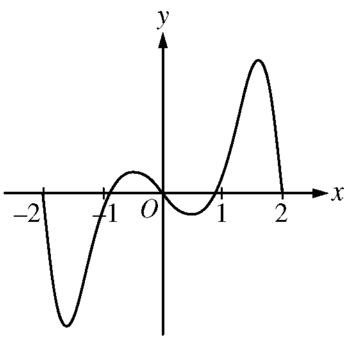
(A)



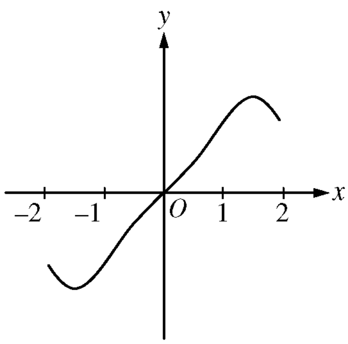
(B)



(C)



(D)



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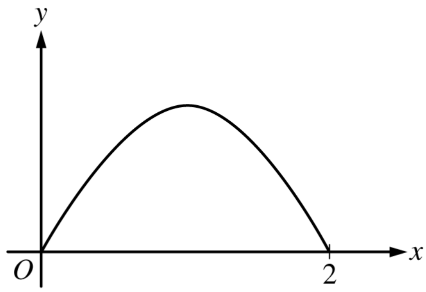
82. The derivative of the function f is given by $f'(x) = e^{-x}\cos\left(x^2\right)$, for all real numbers x. What is the minimum value of $f(x)$ for $-1 \leq x \leq 1$?

(A) $f(-1)$

(B) $f(-0.762)$

(C) $f(1)$

(D) There is no minimum value of $f(x)$ for $-1 \leq x \leq 1$.



83. The base of a solid is the region bounded by a portion of the graph of $y = \sin\left(\frac{\pi}{2}x\right)$ and the x-axis, as shown in the figure above. For the solid, each cross section perpendicular to the x-axis is a rectangle of height 3. Which of the following expressions gives the volume of the solid?

(A) $\int_0^2 3\sin\left(\frac{\pi}{2}x\right)\,dx$

(B) $\int_0^2 3\sin^2\left(\frac{\pi}{2}x\right)\,dx$

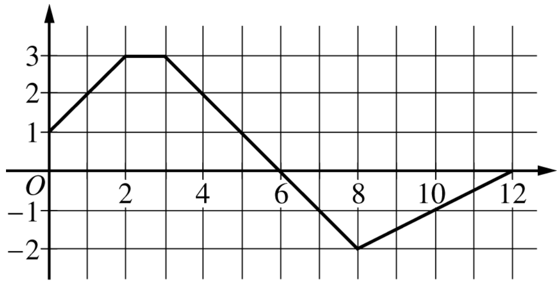
(C) $\int_0^2 3\pi\sin\left(\frac{\pi}{2}x\right)\,dx$

(D) $\int_0^2 3\pi\sin^2\left(\frac{\pi}{2}x\right)\,dx$

GO ON TO THE NEXT PAGE.

84. If g is a twice-differentiable function, where $g(1) = 0.5$ and $\lim_{x\to\infty} g(x) = 4$, then $\int_1^\infty g'(x)\,dx$ is

(A) -3.5 (B) 3.5 (C) 4.5 (D) nonexistent



Graph of f

85. The graph of the function f is shown above. If g is the function defined by $g(x) = \int_2^x f(t)\,dt$, what is the value of $g(10) \cdot g'(10)$?

(A) $\frac{25}{4}$ (B) $\frac{5}{4}$ (C) $-\frac{5}{2}$ (D) $-\frac{25}{2}$

GO ON TO THE NEXT PAGE.

$$f''(x) = x(x-1)^2(x+2)^3$$
$$g''(x) = x(x-1)^2(x+2)^3 + 1$$
$$h''(x) = x(x-1)^2(x+2)^3 - 1$$

86. The twice-differentiable functions f, g, and h have second derivatives given above. Which of the functions f, g, and h have a graph with exactly two points of inflection?

(A) g only

(B) h only

(C) f and g only

(D) f, g, and h

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87. The velocity vector of a particle moving in the xy-plane has components given by $\frac{dx}{dt} = \sin(t^2)$ and $\frac{dy}{dt} = e^{\cos t}$. At time $t = 4$, the position of the particle is $(2, 1)$. What is the y-coordinate of the position vector at time $t = 3$?

(A) 0.410 (B) 0.590 (C) 0.851 (D) 1.410

GO ON TO THE NEXT PAGE.

88. The function f is increasing on the interval $[1, 3]$ and nowhere else. The first derivative of f, f', is continuous for all real numbers. Which of the following could be a table of values for $f'(x)$?

(A)

x	$f'(x)$
0	–1
1	0
2	2
3	0
4	–2

(B)

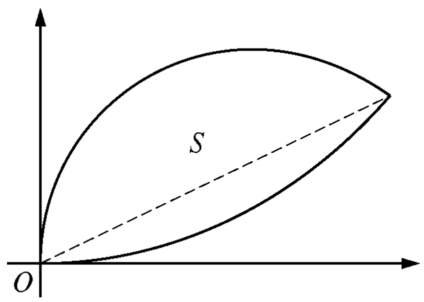
x	$f'(x)$
0	–1
1	1
2	2
3	1
4	–2

(C)

x	$f'(x)$
0	1
1	0
2	1
3	2
4	0

(D)

x	$f'(x)$
0	1
1	0
2	2
3	0
4	–2



89. Let S be the region in the first quadrant bounded above by the graph of the polar curve $r = \cos\theta$ and bounded below by the graph of the polar curve $r = 2\theta$, as shown in the figure above. The two curves intersect when $\theta = 0.450$. What is the area of S ?

(A) 0.232 (B) 0.243 (C) 0.271 (D) 0.384

---

90. If the infinite series $S = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{n}$ is approximated by $P_k = \sum_{n=1}^{k}(-1)^{n+1}\frac{2}{n}$, what is the least value of k for which the alternating series error bound guarantees that $\left|S - P_k\right| < \frac{3}{100}$?

(A) 64 (B) 66 (C) 68 (D) 70

END OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART B ONLY.**

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND ANSWER QUESTIONS 91–94.

Section II: Free-Response Questions

This is the free-response section of the 2017 AP exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP® Calculus BC Exam

SECTION II: Free Response

2017

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.33%

Part B

Number of Questions
4
Time
1 hour
Electronic Device
None allowed
Percent of Section II Score
66.67%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name

   First letter of your first name

2. Date of birth

   Month Day Year

3. Six-digit school code

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.

   No, I do not grant the College Board these rights.

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During Part B, you may continue to work on the questions in Part A without the use of a calculator.

As you begin each part, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 4NBP4-S

68

CALCULUS BC

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. For $0 \le t \le 8$, a particle moving in the xy-plane has position vector $\langle x(t), y(t) \rangle = \langle \sin(2t), t^2 - t \rangle$, where $x(t)$ and $y(t)$ are measured in meters and t is measured in seconds.

   (a) Find the speed of the particle at time $t = 2$ seconds. Indicate units of measure.

   (b) At time $t = 4$ seconds, is the speed of the particle increasing or decreasing? Explain your answer.

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Continue question 1 on page 5.

(c) Find the total distance the particle travels over the time interval $0 \le t \le 5$ seconds.

(d) At time $t = 8$ seconds, the particle begins moving in a straight line. For $t \ge 8$, the particle travels with the same velocity vector that it had at time $t = 8$ seconds. Find the position of the particle at time $t = 10$ seconds.

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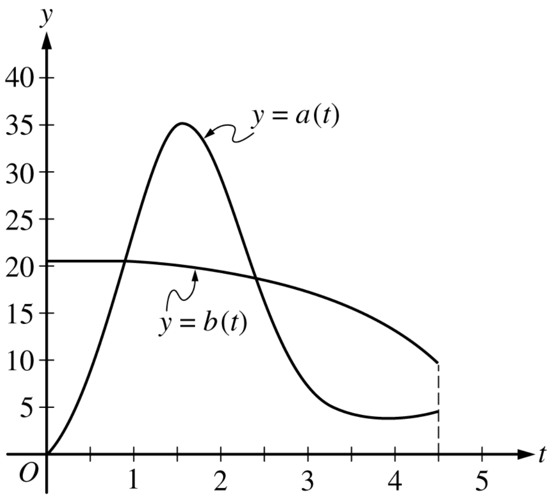

2. During the time interval $0 \le t \le 4.5$ hours, water flows into tank A at a rate of $a(t) = (2t - 5) + 5e^{2\sin t}$ liters per hour. During the same time interval, water flows into tank B at a rate of $b(t)$ liters per hour. Both tanks are empty at time $t = 0$. The graphs of $y = a(t)$ and $y = b(t)$, shown in the figure above, intersect at $t = k$ and $t = 2.416$.

(a) How much water will be in tank A at time $t = 4.5$?

(b) During the time interval $0 \le t \le k$ hours, water flows into tank B at a constant rate of 20.5 liters per hour. What is the difference between the amount of water in tank A and the amount of water in tank B at time $t = k$?

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Continue question 2 on page 7.

(c) The area of the region bounded by the graphs of $y = a(t)$ and $y = b(t)$ for $k \le t \le 2.416$ is 14.470. How much water is in tank B at time $t = 2.416$?

(d) During the time interval $2.7 \le t \le 4.5$ hours, the rate at which water flows into tank B is modeled by $w(t) = 21 - \dfrac{30t}{(t-8)^2}$ liters per hour. Is the difference $w(t) - a(t)$ increasing or decreasing at time $t = 3.5$? Show the work that leads to your answer.

GO ON TO THE NEXT PAGE.

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC

SECTION II, Part B

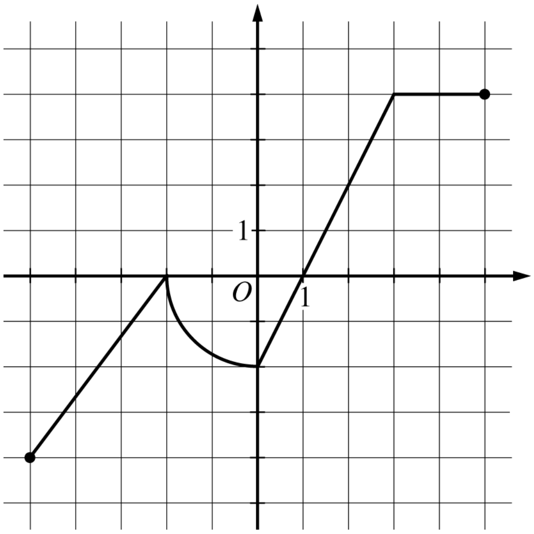
Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED



Graph of f

3. The graph of the function f, consisting of three line segments and a quarter of a circle, is shown above. Let g be the function defined by $g(x) = \int_1^x f(t)\, dt$.

(a) Find the average rate of change of g from $x = -5$ to $x = 5$.

(b) Find the instantaneous rate of change of g with respect to x at $x = 3$, or state that it does not exist.

Continue question 3 on page 15.

NO CALCULATOR ALLOWED

(c) On what open intervals, if any, is the graph of g concave up? Justify your answer.

(d) Find all x-values in the interval $-5 < x < 5$ at which g has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

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NO CALCULATOR ALLOWED

x	0	1	2	3	4	5	6
$f'(x)$	4	3.5	2	0.8	1.7	5.8	7

4. The function f satisfies $f(0) = 20$. The first derivative of f satisfies the inequality $0 \le f'(x) \le 7$ for all x in the closed interval [0, 6]. Selected values of f' are shown in the table above. The function f has a continuous second derivative for all real numbers.

(a) Use a midpoint Riemann sum with three subintervals of equal length indicated by the data in the table to approximate the value of $f(6)$.

(b) Determine whether the actual value of $f(6)$ could be 70. Explain your reasoning.

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Continue question 4 on page 17.

NO CALCULATOR ALLOWED

(c) Evaluate $\int_2^4 f''(x)\,dx$.

(d) Find $\lim_{x \to 0} \dfrac{f(x) - 20e^x}{0.5f(x) - 10}$.

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NO CALCULATOR ALLOWED

5. Consider the differential equation $\frac{dy}{dx} = -1 + \frac{y^2}{x}$.

(a) Show that $\frac{d^2y}{dx^2} = \frac{2y^3 - y^2 - 2xy}{x^2}$.

(b) Let $y = g(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = -1 + \frac{y^2}{x}$ with initial condition $g(4) = 2$. Does g have a relative minimum, a relative maximum, or neither at $x = 4$? Justify your answer.

Continue question 5 on page 19.

(c) Let $y = h(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = -1 + \frac{y^2}{x}$ with initial condition $h(1) = 2$. Write the second-degree Taylor polynomial for h about $x = 1$.

(d) For the function h given in part (c), it is known that $\left|h'''(x)\right| \le 60$ for all x in the interval $0.9 \le x \le 1.1$. Let A represent the approximation of $h(1.1)$ found by using the second-degree Taylor polynomial for h about $x = 1$ from part (c). Use the Lagrange error bound to show that A differs from $h(1.1)$ by at most 0.01.

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NO CALCULATOR ALLOWED

6. Let f be the function defined by $f(x) = \dfrac{1}{x^2 + 9}$.

(a) Evaluate the improper integral $\int_3^{\infty} f(x)\,dx$, or show that the integral diverges.

(b) Determine whether the series $\sum_{n=3}^{\infty} f(n)$ converges or diverges. State the conditions of the test used for determining convergence or divergence.

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Continue question 6 on page 21.

NO CALCULATOR ALLOWED

(c) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\left(e^n \cdot f(n)\right)} = \sum_{n=1}^{\infty} \frac{(-1)^n\left(n^2+9\right)}{e^n}$ converges absolutely, converges conditionally, or diverges.

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STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE FRONT COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**